Joint Optimal Ordering and Weather Hedging Contract Decisions: a Newsvendor Model

YEUNG Yun Sing Samson

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論文摘要

這篇論文考慮傳統的報童問題 (newsvendor model) 的伸延，當中的貨品需求 (order quantity) 與天氣溫度線性地聯繫著。為了對沖非劇變的天氣風險 (non-catastrophic weather risk) 及極大化公司的期望效用 (expected utility)，報童(newsvendor) 不單要決定訂單數量, 而且要考慮是否採用天氣風險對沖策略 (weather risk hedging strategy)。假設天氣期權的價格是外給的。在字典優化目標 (lexicographic optimization objective) 和平均方差優化目標 (mean-variance framework) 下、最優的訂單數量和期權股的數量是同步決定的。以傳統的報童問題作為標準、我們發現如果天氣期權滿足某些條件, 而報童採用那些期權，報童的風險能夠成功地被對沖。

Abstract

This thesis considers an extension of the standard newsvendor model with weather-sensitive product demand, in which the demand depends on the temperature index linearly. In order to hedge the non-catastrophic weather risk and maximize the firm's expected utility, the newsvendor decides not only the order quantity, but also a weather risk hedging strategy with temperature options. The option price is assumed to be exogenous. Optimal order quantity and number of shares of weather options are jointly determined under a lexicographic optimization objective and mean-variance framework. Taking the standard newsvendor model (without weather risk hedging) as the reference, it is shown that the newsvendor's risk can be hedged by giving the newsvendor opportunity to use options.

Keywords: Newsvendor problem, non-catastrophic weather risk hedging, lexicographic optimization, mean-variance framework

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Chapter 1

Introduction

Firms are exposed to a wide variety of risks such as uncertainties about demand for products and supply of key inputs, exchange rate risks, political instabilities, and labor disruptions. Corporate risk management programs aim to systematically manage such risk exposures to increase firm value. This project is concerned with weather risk, the uncertainty in cash flow and earnings caused by weather volatility, or the financial exposure that a business may have to weather events.

Sophisticated firms now use weather hedge contracts to hedge against the financial impact of adverse weather, evening out their weather sensitive earnings. This thesis tries to address impacts of weather risk on operation management in general, and joint determination of weather hedging and operational decisions in particular. Specifically, we consider a risk averse firm selling a product whose demand is weather sensitive and uncertain. The firm can acquire weather hedge contracts to reduce its risk exposure. In addition, it needs to jointly determine the production/inventory quantity and the weather hedging strategy to maximize the firm's expected utility, where the hedging strategy encompasses the type(s) of hedge contract(s) and the hedging amount of each type, and the utility may be attained by means of mean-variance framework.

The objective of this thesis is to analyze the joint treatment of operational decision and weather hedging contract decision. The aim is to minimize the non-catastrophic weather risk or, on the other hand, maximize the firm's expected utility, for the product with weather-sensitive demand.

We will first study the interactions between operational and weather risk strategies of risk averse firms within a stylized, but representative modeling setting; i.e., a *newsvendor* context. In this setting, only a single type of hedge contract is considered. An interesting issue is whether a firm should order/produce more with a weather risk hedge contract than without. Under lexicographic optimization objective, it can be shown that the profit variance can be minimized with the same optimal order quantity and same maximum expected profit with the weather risk hedge contract, while under mean-variance optimization, a numerical example shows that a firm should order more with a weather risk hedge contract. Useful extensions include general demand function, dependency of newsvendor parameters and multiple types of weather contracts (discussed in Chapter 8).

These problems are largely motivated by the concerns of distributors or manufacturers. We expect that the proposed research will contribute to both areas of operations management and weather hedging by way of a set of models and solution methods, which, on the other hand, will lead to a significant academical advancement, and on the other will aid various operations/supply-chain decisions so as to improve earnings through better weather risk management. The models will provide a sound understanding of the interactions between weather risk management and production/inventory decisions so that decision makers can analyze, evaluate, and implement decision rules. Integrating risk management into operations decisions also represents an emerging area of research in supply chain management.

This thesis is organized as follows: in Chapter 2, we discuss the feasibility of weather derivative in Hong Kong and the types of weather risk. In Chapter 3, though the integrated treatment of weather risk management and production decisions does not exist in the literature, some related studies are reviewed. On the other hand, the literature of weather option pricing will also be discussed. In Chapter 4, a basic model of the newsvendor problem with a weather option will be established. In Chapter 5, we analyze the newsvendor model and discover some features of the options for higher degree of profit variance reduction. In Chapter 6, we analyze the model under lexicographic objective (objective 1). We formulate a multiple objective decision model for the newsvendor. We assume that the newsvendor ranks her objectives lexicographically, i.e., she ranks them in terms of importance and optimizes the objectives sequentially. The objective on higher priority is the maximization of expected profit and that on lower priority is the minimization of profit variance. Some numerical examples will be carried out. In Chapter 7, we analyze the model under mean-variance framework (objective 2). We compare the mean-variance function value of the newsvendor with and without option. A numerical result displays that a firm should order more products with a weather risk hedge contract than without. The last chapter concludes the thesis with a brief summary and the discussion of a potentially useful extension.

Chapter 2

Background

In essence, weather represents just a special case of uncontrollable processes that affect firm's revenues. Our models are largely motivated by the concerns of companies, which are related to the problems to be investigated. Consider the following three examples.

(1) Although the profits derived from the retail sale of clothing are also affected by exchange rates, tariffs, and the cost of raw materials and labor, the weather is recognized as having a major influence on sales figures. A retailer chain once attributed a 65% plunge in its quarter earnings to a cold wet May (Sanford, 2002). In 2002, Giordano, a Hong Kong based apparel store chain, also blamed the unfavorable weather for its lower sales (Lee et al., 2002). (2) Hallmark found that unfavorable weather during the last few days before Valentine's Day affects the independent retailers which sell its greeting cards, hitting Hallmark the next year when they order fewer cards because of the prior year's poor sales (Zolkos, 2001). (3) To meet customer demand, a propane/heating oil distributor must purchase adequate inventory to cover an

expected heating season. If the season is warmer than normal, sales volume will decrease, which will leave the wholesaler with excess inventory and the associated storage expenses.

From these examples, we can see that weather risk is distinctive from commodity price risk, because the foremost weather impact is on the *volume* (i.e., the demand or output of the concerned product). In contrast, the *direct effect* of commodity price or exchange rate is on the margin, or in other words, the cost. Though other sources of risk like fashion risk also affect demand, weather risk can now be hedged financially.

According to an estimate by the US National Research Council, 46% of US GDP are affected by weather (Lustgarten, 2004). Sophisticated firms now use weather contracts or derivatives, such as options, futures, or combinations of both, to hedge against the financial impact of adverse weather, which evens out their weather sensitive earnings. (An example of option contract will be described in chapter 4.) Weather risk derivatives or contracts are traded both in exchanges and over the counter. Although they were only introduced on Chicago Mercantile Exchange (CME) in 1997, the volume of transacted weather hedge derivatives reached $4.6 billion during April 2003 and March 2004. A recent survey by the Weather Risk Management Association (WRMA), the international trade organization of the weather risk management industry, found that over one third of the hedge-product buyers are from Asia (WRMA, 2004). In the Asia-Pacific region, weather hedge products are traded in Japan, South Korea, Australia, New Zealand and Taiwan. It is possible that weather products will be traded in Hong Kong in the near future (discussed in chapter 2.1).

This development calls for models that integrate weather risk management and production/inventory decision. Such models should be able to account for impacts of weather on product demand, weather hedge strategy, risk attitude, production/inventory costs. Our proposed research, though not aiming to tackle all of these issues, presents an effort towards this direction. Weather risk management and production/replenishment decisions should and can be treated jointly. Such treatment does not seem to exist in the literature. However, there are related studies.

2.1 Applicability of Weather Derivative in Hong Kong: The Recreation Industry

Although many industries in Hong Kong are affected by weather similar to elsewhere, the tourism industry is probably the most vulnerable. Hong Kong's economy relies heavily on the tourism industry, and the tourism industry, as part of the recreation industry, is often affected by weather volatility. Bad weather can deter people from going outdoors, thus the revenues for public transportation, retailers, theme parks and other tourism-related business, will decrease. For theme parks in Hong Kong, such as the Ocean Park or the up-coming Hong Kong Disneyland, weather conditions greatly affect park attendance and therefore influence the revenue stream. Traditionally the weather risks were considered non-diversifiable and beyond human control. Some kinds of weather insurance may have been possible but they were too costly and the insured weather event were not highly correlated with the revenue stream. Therefore it makes good sense to develop a comprehensive risk management strategy to enable theme parks in Hong Kong to diversify their weather risk. The forming of such strategy requires a pricing methodology of weather derivatives with proper underlying weather events, together with alternative weather risk control tools and certain regulatory concerns.

While designing the weather option, we notice that the weather variable should be carefully selected so that its influence on the revenue stream can be shown clearly. To satisfy the needs of theme parks in Hong Kong, this weather product should be customized; therefore the climate of Hong Kong should be studied to analyze theme parks' needs.

According to Hong Kong Observatory's resources, we can identify the characters of the climate in Hong Kong. Hong Kong's climate is sub-tropical, tending towards temperature for nearly half the year. Severe weather phenomena that can affect Hong Kong include tropical cyclones, strong winter monsoon winds, and thunderstorms with associated squalls that are most frequent from June to October. In this time period, September is the month in which Hong Kong is most likely to be affected by tropical cyclones, although gales are quite common at any time between May and November. Moreover, June and October is the time period recorded as the highest park attendance period for Ocean Park, and most probably for other up-coming theme parks in Hong Kong as well, such as the Hong Kong Disneyland. The matching of time period between severe weather phenomena and park attendance fluctuation indicates the need of hedging with a weather product.

However, one shortcoming is that it will be difficult to identify the complicated factors affecting the visitor flow to theme parks without the understanding of theme park management and operational analysis. Another shortcoming is that the customer's preference is partly influenced by variety-seeking and seasonality effects. A further shortcoming is that the visitor flow data obtained is limited. For a risk management team of a theme park, a full history of park attendance number may reveal more useful information to more accurately perform the weather risk management strategy better.

The derivative risk management tool can be used together with a further analysis of the revenue stream and other risk-control mechanisms. As more weather risk management tools enter the market, theme park will be able to choose among different weather indexes to perform the analysis and the weather risk can be more easily hedged.

We concluded that in Hong Kong, excessive rainfall, thunderstorms and typhoons can be more suitable underlying weather events than temperature fluctuations. Unlike in U.S., where the temperature (HDD/CDD) contracts (Li, 2004) are actively traded in the CME to meet the needs for the energy sector, the temperature fluctuations in Hong Kong in the different seasons do not influence the recreation industry so much as rainfall amount and typhoons. (Remember other sectors in Hong Kong such as apparel industry (e.g. Giordano) are also subject to weather risk.)

2.2 Types of Weather Risk

Weather phenomena have significant effects on the value generation prospects of any economic activity. Different aspects of weather phenomena range from temperature levels, humidity levels, precipitation levels to hurricanes and tornadoes. Weather risk is the uncertainty of cash flow caused by such weather events. The energy sector, e.g. heat or gas provider, and the recreation industry, e.g. theme parks and recreational product makers, whose profits depend heavily on weather conditions, are directly exposed to weather risks. As such, weather derivatives offer these companies the chance to lessen the weather risk and ease the economic consequences.

There are basically two types of weather risk, insurable weather risk and uninsurable weather risk. Different approaches should be taken to mitigate different weather risk exposures. Note that not all the business risks arising from adverse weather conditions can be fully or even partially hedged or insured against.

The first type of weather risk, the insurable weather risk, includes mostly extreme weather events, such as tornadoes, floods and hurricanes. Business losses arising from these extreme weather events - such as a tornado shutting down power in a certain district - cannot be hedged against using a weather derivative, but some form of business interruption insurance and catastrophic insurance can be helpful in these unpredictable situations. Although these extreme weather events are rare, many companies have long purchased insurance policies to protect themselves against large losses resulting from these meteorological events. In this situation a company identifies the catastrophic weather events that have an impact on its revenue stream and arranges catastrophic insurance coverage. The insurance companies carefully evaluate the risk probability and set an appropriate premium.

The other type of weather risk is non-catastrophic, but it still has an impact on the revenue generating prospect of a company. These weather risk talks about adverse weather events such as severe and continuous precipitation, rainstorms or typhoons. This type of risk mitigation seeks to provide protection against fluctuation in the revenue stream deviating from the norm and would employ a weather derivative as hedge. It has only been within the past decade that derivatives have allowed companies to hedge against weather that is not necessarily catastrophic, but which could still devastate regular earnings. In this case it is important to note the effect of weather events on the value generation prospects of any economic activity. The impact of the weather event and the related economic activity would affect the construction of a hedging portfolio involving weather derivatives.

In this thesis, we focus on the application of weather derivatives on non-catastrophic weather risks.

Chapter 3

Literature Review

In the literature of weather risk management, a firm that buys a weather hedge product (or derivative) is assumed to answer two questions: What is the relationship between weather and net revenue. What is the likelihood that weather will occur such that earnings fall to an unacceptable level? The first question relates to how much protection is needed, while the second relates to weather index level (e.g., temperature) for which protection is desired. See Dischal (2001), and Harrington and Niehaus (2003). However, little attention has been devoted to the interaction between weather risk and operations management. For example, consider a newsvendor facing a weather-sensitive demand. His net revenue will be jointly determined by the initial order quantity and the overall weather in the season. The hedged net revenue depends clearly on both ordering decision and weather hedging structure.

There is a fast-growing body of literature on weather hedge product pricing which involves the calculation of the estimate of expected payoff; see

Jewson and Rodrigo (2002), and Jewson (2003). Weather Derivatives are classic examples of incomplete markets. As the underlying weather variables are usually very illiquid and even not replicable, the standard 'risk-neutral' point of view is not applicable to evaluate the derivatives based on weather variables. Therefore, a direct application of the standard derivative pricing theory, based on the no-arbitrage and market completeness assumptions, is inadequate.

In addition, although weather derivatives share features with options and futures, the structures are not identical. The statistical processes followed by temperatures or rainfall amounts are quite different from those governing price movements. There have been many previous works about the pricing of weather products. Figlewski and Levich (2002) and German (1999) have proposed several pricing and simulation methods for catastrophic bonds and weather instruments. Pricing of a weather derivative for non-catastrophic weather risk is generally carried out following one of the following procedures:

1. Utility Optimization Method

Pauline and Nicole (2002) determine the optimal structure of derivatives written on an illiquid asset, such as a catastrophic or a weather event. The modeling for the optimal design of such derivatives involves the definition of a choice criterion for the different agents. For simplicity, and the agents, the bank and the investor, are assumed to be risk averse and to have an exponential utility criterion. The bank wants to hedge its position at maturity for exposure to a non-financial risk. The bank sells a contract to the investor by choosing the optimal structure of this contract according to its utility. On the other hand, the investor finds the transaction interesting only when its expected utility is the same whether the investor buys the contract or not. The optimal structure can be determined by maximizing the bank's expected utility under the constraint that the investor's expected utility is unharmed.

2. Expected discounted value approach

Since there is no liquid market in these contracts, Black-Scholes style pricing is not entirely satisfactory. Mark Davis (2001) and Brody, Syroka and Zervos (2002) suggested that valuation of weather derivatives is generally conducted on an 'expected discounted value' basis, discounting at the risk-free rate but under the physical measure of the weather variable. The exposure or loss for each outcome is estimated and a corresponding probability of occurrence is obtained from a sample of historical events. When the pricing method is quite straight forward, the empirical/statistical distribution of the underlying weather variable is essential to make the discounting process accurate. Therefore model building and estimation of the weather variable is very important in this method. In this thesis, the exogenous fair value of the weather option is assumed to be priced under this approach.

3. Option Pricing Theory

It is assumed that a valuation technique similar to that employed for pricing options and other claims on marketable assets, such as stocks and bonds, can be used (e.g., Black-Scholes pricing formula). The critical distinction between pricing an ordinary stock derivative and a weather derivative is that the underlying is not tradable in our problem, which makes it impossible to construct a replicating portfolio. Cao and Wei (2000) suggested that although the assumptions under this valuation method do not hold, a proxy market asset can be used for replication if possible. The idea behind this method is that if we can find a suitable proxy asset, we can mimic the value dynamics of the weather variable and evaluate it. The problem is whether such a proxy is feasible and reasonable.

From the financial perspective, this literature provides the foundation for risk hedging.

The research on operational hedging is enormous. The term "operational hedging" set apart from "financial hedging" as follows: The latter is realized through financial tools, such as options and swaps, while the former manages a firm's risk operationally, such as delaying the production decisions until after more accurate demand information is acquired., or flexible capacity to better match supply and demand. Operational hedging is mostly obtained by real options - opportunities to delay and adjust investments and operating decisions over time in response to resolution of uncertainty (Triantis, 2000).

Most of the studies in this literature focuses on real options in a global supply chain context. Examples of real options studied include switching production and sourcing strategies contingent on demand and exchange rate uncertainties, and switching among supply chain network structures (Cohen and Huchzermeier, 1999). In a recent survey, Van Mieghem (2003) reviews the literature on strategic and adjustments under uncertainty, both with and without financial hedging. In another recent survey, Boyabatl and Toktay (2004) provide an excellent summary and critique on the existing literature on operational hedging.

The work of Ding and Kouvelis (2001) is the one closest in spirit to this project. They study the interaction of operational and financial hedging policies of a risk averse global firm facing demand and exchange rate uncertainities. They consider a two-stage newsvendor model. In the first stage, the firm determines the initial production quantity and amount of exchange rate hedging. In the second stage, after all the uncertainty is resolved, a further production allocation decision (e.g., how many units to localize and distribute to the foreign market) is made to mazimize firm's utility, which incurs additional costs. The second stage allocation decision is the firm's real option that provides it an operational hedge against the demand and exchange rate uncertainties. However, the exchange rate risk is associated essentially with price uncertainties, which ours mainly associated with volume uncertainties. Moreover, there is no real option in our models.

Gaur and Seshadri (2004) address the problem of hedging inventory risk in a newsvendor setting where the product demand is correlated with the price of a financial asset which is tradable. They derive optimal hedging transactions that minimize the variance of profit and increase the expected utility for a risk-averse decison maker. They show that for a wide range of hedging strategies and utility functions, a risk-averse decision maker orders more inventory when she hedges the inventory risk. While Caldentey and Haugh (2003) address the optimal joint policies for financial hedging and operations, such as production/inventory decisions, the firm as an asset is traded continuously in the financial market and can get self-financed. In our models, the weather index is however not an "asset".

In summary, the problem of integrated treatment of weather risk management and production decision that we investigate have not been addressed in the literature.

Chapter 4

Basic Model

This model is largely motivated by the case study of Dischel and Barrieu (2001), which can be highlighted as follows. A firm sells a seasonal product (e.g., propane) with temperature-sensitive demand. It wants to buy call options on temperature to hedge its exposure. The call option is priced at $1 million by a financial institution that will pay $40,000 for each 0.1 C that the season's average temperature is above a strike level, with a cap of $4 million. thus, if the season's average temperature is above turns out to be 0.5 C above the strike level, then the firm will receive $2 million; while if it is 1.2 C higher, then it only receives the cap amount. If the average temperature is 0.2 C below the strike level, then the firm receive no compensation. If the firm wants to spend less to hedge, then it can buy any fraction of this option, e.g., a premium of $0.5 million will pay the firm only $20,000 per 0.1 C, with a cap of $2 million. the risk averse firm needs to determine the quantity of call options to optimize its expected utility. One can further infer from the case context that the initial inventory ordering quantity is also a decision variable. Such a scenario often arises in retailing and manufacturing (Malinow (2002), XL Weather & Energy (2004)).

Suppose that a newsvendor sells a seasonal product whose demand during the season is contingent on the weather, such as the average temperature. The favorable weather generates strong sales and hence high revenue, while unfavorable weather significantly shrinks the demand as well as revenue, and sometimes even causes big losses to the newsvendor.

To protect his revenue, the newsvendor can buy a weather derivative in the over-the-counter (OTC) weather risk market that will pay him if the weather is unfavorable and thereby compensate for weak revenue. The level of compensation depends on the premium that the newsvendor pays. The actual profit depends on the initial order quantity, the weather, the actual demand and the compensation level. The problem is to decide on the amount of premium and initial order quantity so that the firm's expected utility is optimized.

It should be noted that in this thesis, we exemplify the weather index as temperature, which is without loss of generality.

4.1 Notations

Denote by $x(t)$ the random demand following a density function $g(x)$ parameterized by the weather index t following density function $f(t)$(say, average temperature). The strike level is agreed to be t^*. If the newsvendor pays a premium of K, then he will receive $i(K)$ for each unit of deviation $(t - t^*)$, but the total amount is capped at $\hat{C}(K)$. If the realized index value is less than t^*, then the newsvendor receives no payment. This is a kind of call option (Hull, 2002).

For each unit of sale, s is received. The pre-season purchasing cost is c per unit of product. Excess demand will be lost. (While in the case study mentioned above, the firm is obliged to satisfy all the demand.) In an extreme cold winter, demand will be high, and so will the spot price. However, gas firms typically want to protect revenue against mild winters, so we ignore this fact. This also makes it nearly impossible for the firm to sell back to the spot market for profit in the mid of season.) Any leftover at the season end will be salvaged at c_h ($< c$), and the shortage cost is b per unit. In reality, both c_h and b may also depend on t, but we leave this in later future studies.

Table 4.1 summarizes the notations of parameters used in the model. The reason of the existence of revised formulation of net option payoff will be discussed in Chapter 5.

Type	Symbol	Description
Operation Parameters	s	Unit sales price
	c	Unit purchasing cost
	Q	Order quantity (a decision variable with the optimal value Q^*)
	c_h	Unit salvage value
	b	Unit shortage cost
Option Parameters (Original Formulation)	t^*	Strike temperature
	K	Option premium (a decision variable with the optimal value K^*)
	$\hat{C}(K)$	Upper bound of option payoff
	$i(K)$	Marginal incremental parameter
Option Parameters (Revised Formulation)	r	Risk free interest rate
	T	Time to maturity
	N	Number of share(s) of call spread in a long position (a decision variable with the optimal value N^*)
	C_A, C_B	Fair prices of call options A and B respectively
	K_A, K_B	Strike temperature of call options A and B respectively
	P_A, P_B	Marginal incremental parameter of call option A and B respectively
Random Parameters	t	Random temperature at time T
	$x(t)$	Random demand at time T
	$f(t)$	P.D.F. of temperature at time T
	$g(x)$	P.D.F. of demand at time T
	$\pi_1(x)$	Random newsvendor profit at time T
	$\pi_2(t)$	Random net option payoff at time T
	$\pi(t)$	Random total newsvendor profit at time T

Table 4.1: Notations of Parameters.

4.2 Assumptions

Assumption 1 $s > c > c_h \geqslant 0$, $b \geqslant 0$.

Assumption 2 *x is linearly decreasing with t and floored above zero.*

$$x(t) \ = \ (c_x + m_x t)^+. \tag{4.1}$$

where $m_x < 0$

Note that $a^+ := \max[a, 0]$

From (4.1), it is immediate that

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ F(-\frac{c_x}{m_x}) & \text{if } x = 0 \\ f(\frac{x - c_x}{m_x}) & \text{if } x > 0. \end{cases} \tag{4.2}$$

Assumption 3 *The fair value of weather option is exogenous and priced under expected discounted value approach (refer to chapter 3).*

$$\text{fair value} = e^{-rT} E[\text{option payoff}]. \tag{4.3}$$

Remark 1 *If the fair value of weather option is exogenous but not necessarily priced under expected discounted value approach, i.e., (4.3) does not hold, then the expected value of net option payoff will not always equal to zero.*

4.3 The Profit Model

We consider an extended single-period newsvendor problem in which the profit function consists of two parts: profit from sales, π_1 and net option payoff, π_2. We now develop the expression for the profit as a function of option premium K, ordering quantity Q, random temperature t and random demand x.

$$\pi(K, Q, t, x(t)) = \pi_1(Q, x(t)) + \pi_2(K, t). \tag{4.4}$$

where

$$\pi_1(Q, x(t)) = \begin{cases} sx(t) + c_h(Q - x(t)) - cQ & \text{if } x(t) \leqslant Q \\ sQ - b(x(t) - Q) - cQ & \text{if } x(t) > Q. \end{cases} \tag{4.5}$$

$$\pi_2(K, t) = \min\left[\hat{C}(K), i(K)(t - t^*)^+\right] - K. \tag{4.6}$$

where the discount factor e^{-rT} is included in the min[ ] term implicitly, and the option premium K follows (4.3).

Note that (4.4) - (4.6) differ from the respective counterparts in Gaur and Seshadri (2004) and Ding et al. (2004). We can use a replicated portfolio to simplify (4.6) (will be discussed in Chapter 5).

In terms of objective function, we assume that the risk-averse firm will choose to optimize it lexicographically in terms of maximizing expected profit and minimizing profit variance (to be referred to as *Objective 1*). This form of objective is studied by Chen and Parlar (2004). Another commonly used objective is in the mean-variance framework (to be referred to as *Objective 2*); Chen and Federgruen (2000); Gaur and Seshadri (2001); Ding and Kouvelis (2001). (Theoretically, a risk-neutral firm will unlikely engage in weather hedging.)

The main reason for the use of lexicographic optimization is that the problem exists multiple optimal solutions in the objective of expected profit maximization (will be discussed in Chapter 6). The lexicographic optimization is able to deal with this difficulty. For the use of mean-variance optimization, it is reasonable to quantify the utility of a risk averse newsvendor using mean-variance measurement and optimize the mean-variance objective function. The final objective not mentioned in the thesis is to optimize the objective function under VaR criterion; i.e., assume that the risk-averse firm will choose to maximize the probability of exceeding a prespecified target profit level. Although this objective also makes good sense to a risk averse newsvendor, it is technically difficult to solve it. Therefore, VaR criterion will be tackled after some insights have been gained from lexicographic optimization and mean-variance optimization. VaR Optimization is out of the scope of this thesis and will be left as future studies.

The goal is to optimize an objective function by choosing K and Q. It should be clear that for an expected profit maximization objective, optimal K and Q can separately be resolved. But for the above-mentioned two objectives, the optimal solution of K and Q must be obtained concurrently.

When the demand is assumed to be linear in t and to take an additive random term, a preliminary study with objective 2 shows some promising result. Several numerical examples reveal that the introduction of weather protection often leads to bigger order quantities, there are however exception. Therefore, the interplay between the parameters is worthy of in-depth analysis.

Chapter 5

Fundamental Analysis

In this chapter, we first do the analysis on profit from sales, π_1 and net option payoff, π_2 separately. Then, a joint analysis of π_1 and π_2 will be carried out for the reformulation of total newsvendor profit, π. It can be shown that the call spread (will be discussed in Section 5.2) with certain characteristic will result in a higher degree of profit variance reduction.

5.1 Sales Profit Analysis

Without risk consideration, expected profit maximization is the standard newsvendor problem objective. We will first obtain the expression of (Q, K) that maximizes the expected profit, then compare (Q, K) derived under risk consideration.

Taking expectation of (4.5) makes the term $\pi_2(K, t)$ zero under (4.3). Therefore, taking expectation of (4.5) over x and simplifying leads to the well known expected one-period profit function.

$$E_x[\pi_1(Q,x(t))] = (s-c_h)\mu_x + (c_h - c)Q - (s+b-c_h)\int_Q^\infty (x-Q)g(x)dx. \quad (5.1)$$

Equating the first derivative of (5.1) w.r.t. Q to zero, we obtain the optimal order quantity Q^* satisfying the following necessary condition.

$$Q^* = G^{-1}\left(\frac{s-c+b}{s-c_h+b}\right). \quad (5.2)$$

The negative sign of second derivative of (5.1) ensures the maximality of Q^*. Note that the optimal order quantity is with respect to the transformed demand cumulative distribution function, $G(x)$.

Since the newsvendor expected profit with option is the same as the newsvendor expected profit without option,

$$E_t[\pi(Q,K,x(t),t)] = E_x[\pi_1(Q,x(t))]. \quad (5.3)$$

Therefore, maximizing $E_t[\pi(Q,K,x(t),t)]$ is equivalent to maximizing $E_x[\pi_1(Q,x(t))]$, which is independent of K. The optimal option premium K^* can assume any non-negative value.

$$K^* \in [0,\infty). \quad (5.4)$$

5.2 Option Analysis

To quantify the risk of the newsvendor profit, a variance term will be included in the objective function. However, the variance of (4.6) is very complicated to be expressed. It induces the reformulation of (4.6) into standard net option payoff expression.

Consider the following Replicating Portfolio:

- 1 share (long position) of European call option on temperature index with price C_A, strike temperature K_A, and proportionality constant P_A
- -1 share (short position) of European call option on temperature index with price C_B, strike temperature K_B, and proportionality constant P_B

where $K_B > K_A$, $P_A \geqslant P_B > 0$ and $C_A > C_B > 0$, and C_A, C_B follow (4.3).

$$C_i = e^{-rT}E_t[P_i(t-K_i)^+], i = A, B. \tag{5.5}$$

We have the relationship between the prices of different call options.

$$C_A > C_B \text{ if } K_B > K_A \text{ and } P_A \geqslant P_B > 0\ . \tag{5.6}$$

Proof.

$$\frac{\partial C}{\partial K} = Pe^{-rT}\frac{\partial E_t[(t-K)^+]}{\partial K} < 0 \text{ for } P > 0$$

$$\frac{\partial C}{\partial P} = e^{-rT}E_t[(t-K)^+] > 0$$

Q.E.D.

We proceed with the cash flow analysis of the portfolio.

- At time 0, cash flow $= -(C_A - C_B) < 0$
- At time T, cash flow $= P_A(t - K_A)^+ - P_B(t - K_B)^+ \geqslant 0$
- Net Future Value $= -(C_A - C_B)e^{rT} + P_A(t - K_A)^+ - P_B(t - K_B)^+$

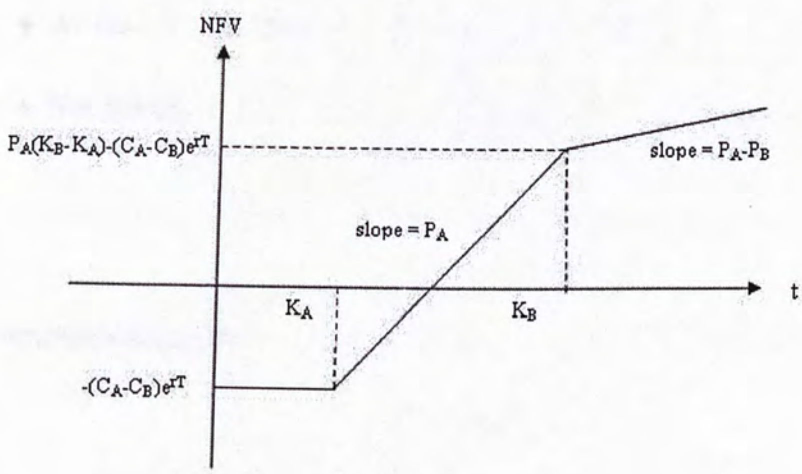

Figure 5.1: NFV against t with one share of call spread.

We call this portfolio (including one share of call option in a long position and one share of call option in a short position) as call spread. For the above payoff pattern, it is equivalent to one share of call spread in a long position. The number of shares of call spread can be changed to adjust the pattern of the curve.

Modified Replicating Portfolio

- N share (long position) of European call option on temperature index with attributes (C_A, K_A, P_A)
- -N share (short position) of European call option on temperature index with attributes (C_B, K_B, P_B)

Cash Flow Analysis:

- At time 0, cash flow $= -N(C_A - C_B) < 0$
- At time T, cash flow $= N[P_A(t-K_A)^+ - P_B(t-K_B)^+] \geqslant 0$
- Net Future Value $= -N(C_A - C_B)e^{rT} + N[P_A(t-K_A)^+ - P_B(t-K_B)^+]$

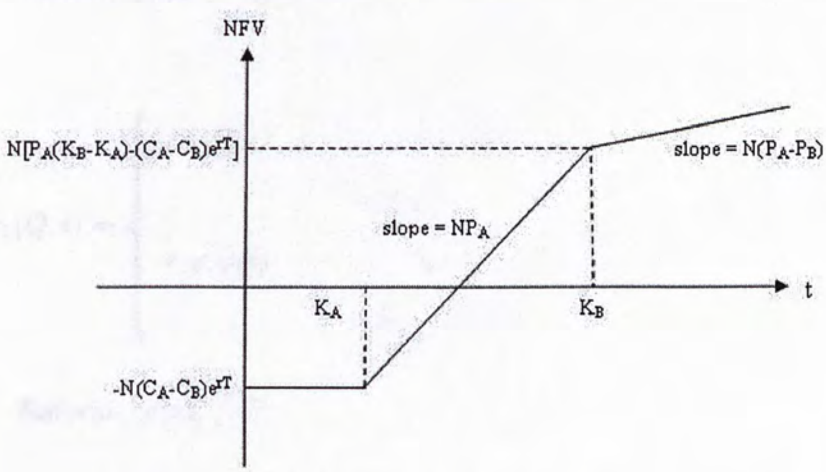

Figure 5.2: NFV against t with N shares of call spread.

The payoff pattern of the call spread can replicate the option payoff pattern appeared in the original formulation.

The Net Portfolio Payoff

$$\pi_2(N,t) = N[-(C_A - C_B)e^{rT} + P_A(t-K_A)^+ - P_B(t-K_B)^+] \qquad (5.7)$$

Note that the decision variable changes from K to N, the number of shares of call spread in a long position at time 0 (the time that the newsvendor make the order quantity decision).

5.3 Profit Function Reformulation

Since π_1 is a function of x, and π_2 is a function of t. x and t are interrelated. We reformulate (4.4) as a function of t for facilitating the direct treatment of π_1 and π_2.

Denote $t_1(Q) := \frac{Q-c_x}{m_x}$

$$\pi_1(Q,t)=\begin{cases} \pi_{11}(Q,t) := (s+b-c)Q-bc_x \\ \qquad -bm_xt & \text{if } \underline{t}\leqslant t< t_1(Q) \\ \pi_{12}(Q,t) := (s-c_h)c_x+(c_h-c)Q \\ \qquad +(s-c_h)m_xt & \text{if } t_1(Q)\leqslant t\leqslant \bar{t}. \end{cases} \tag{5.8}$$

Reformulating (5.7)

$$\pi_2(N,t)=\begin{cases} \pi_{21}(N) := N[-(C_A-C_B)e^{rT}] & \text{if } \underline{t}\leqslant t\leqslant K_A \\ \pi_{22}(N,t) := N[-(C_A-C_B)e^{rT} \\ \qquad +P_A(t-K_A)] & \text{if } K_A\leqslant t\leqslant K_B \\ \pi_{23}(N,t) := N[-(C_A-C_B)e^{rT} \\ \qquad +P_BK_B-P_AK_A \\ \qquad +(P_A-P_B)t] & \text{if } K_B\leqslant t\leqslant \bar{t}. \end{cases} \tag{5.9}$$

Lemma 1 *The weather option with net option payoff π_2 can reduce the newsvendor profit variance if and only if $V_t[\pi_2] + 2cov[\pi_1, \pi_2] < 0$. Moreover, the more negative the term $V_t[\pi_2] + 2cov[\pi_1, \pi_2]$, the higher the profit variance reduction.*

Proof.

$$\begin{aligned} V_t[\pi_1 + \pi_2] &= V_t[\pi_1] + V_t[\pi_2] + 2cov[\pi_1, \pi_2] \\ \Leftrightarrow V_t[\pi_1] - V_t[\pi_1 + \pi_2] &= -V_t[\pi_2] - 2cov[\pi_1, \pi_2] \end{aligned}$$

To reduce the newsvendor profit variance, i.e.

$$\begin{aligned} V_t[\pi_1] &> V_t[\pi_1 + \pi_2] \\ \Leftrightarrow V_t[\pi_2] + 2cov[\pi_1, \pi_2] &< 0 \end{aligned} \tag{5.10}$$

Q.E.D.

(5.10) can be rearranged as

$$cov[\pi_1, \pi_2] < -\frac{V_t[\pi_2]}{2} = -\frac{E_t[\pi_2^2]}{2} < 0$$

As a result, (5.10) holds when π_1 and π_2 are negatively correlated and the covariance is smaller than the negative of halved second moment of π_2. The weather option in the derivative market should possess some characteristics for (5.10) to hold such that the profit variance can be reduced. This induces the following proposition.

Proposition 1 *In order to achieve the purpose of weather risk hedging, that is to minimize the profit variance, the condition $K_A \geqslant t_1(Q^*)$ must be hold,* *where Q^* is the optimal order quantity and the value is different in different optimization objective. If no such an weather option exist in derivative market, then the newsvendor should not engage in the weather hedging strategy.*

Proof.

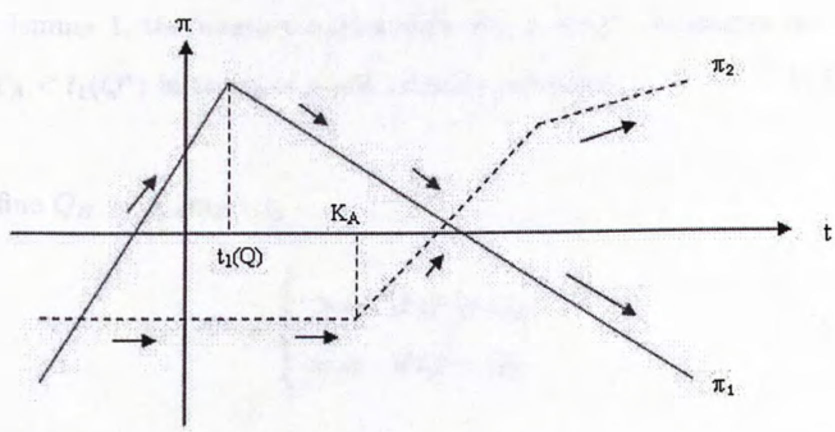

Figure 5.3: Correlation between π_1 and π_2 with $K_A \geqslant t_1(Q^*)$.

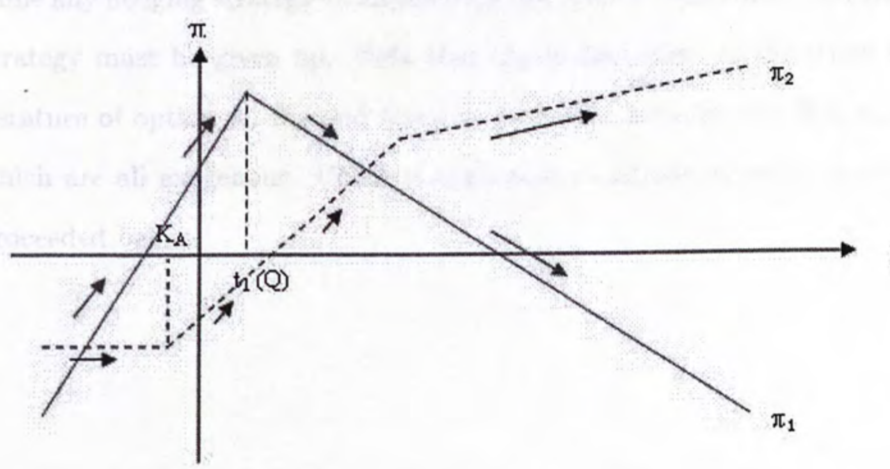

Figure 5.4: Correlation between π_1 and π_2 with $K_A < t_1(Q^*)$.

As depicted from Figure 5.3 and Figure 5.4, the covariance between π_1 and π_2 with $K_A \geqslant t_1(Q^*)$ is smaller than the covariance with $K_A < t_1(Q^*)$

$$cov[\pi_1, \pi_2]_{K_A \geqslant t_1(Q^*)} < cov[\pi_1, \pi_2]_{K_A < t_1(Q^*)} \tag{5.11}$$

By lemma 1, the weather option with $K_A \geqslant t_1(Q^*)$ dominates the one with $K_A < t_1(Q^*)$ in terms of profit variance reduction. Q.E.D.

Define $Q_H := K_A m_x + c_x$

$$N^* = \begin{cases} \geqslant 0 & \text{if } Q^* \geqslant Q_H \\ = 0 & \text{if } Q^* < Q_H. \end{cases} \tag{5.12}$$

Q_H is called the hedging point. In orders, if the optimal order quantity Q^* is larger than or equal to the hedging point Q_H, the newsvendor can assume any hedging strategy to improve his risk profile. Otherwise, the hedging strategy must be given up. Note that Q_H is dependent of the strike temperature of option A, K_A and linearity constants between $x(t)$ & t, c_x, m_x, which are all exogenous. (5.12) is applicable to all optimization objectives proceeded below.

By Proposition 1, the newsvendor total profit can be expressed as follows.

$$\pi(Q,N,t)=\begin{cases}\pi_{11}(Q,t)+\pi_{21}(N) & \text{if } \underline{t}\leqslant t<t_1(Q)\\ \pi_{12}(Q,t)+\pi_{21}(N) & \text{if } t_1(Q)\leqslant t<K_A\\ \pi_{12}(Q,t)+\pi_{22}(N,t) & \text{if } K_A\leqslant t<K_B\\ \pi_{12}(Q,t)+\pi_{23}(N,t) & \text{if } K_B\leqslant t<\bar{t}.\end{cases} \tag{5.13}$$

Figure 5.5 depicts the total newsvendor profit pattern for each realization of t with $K_A \geqslant t_q(Q)$

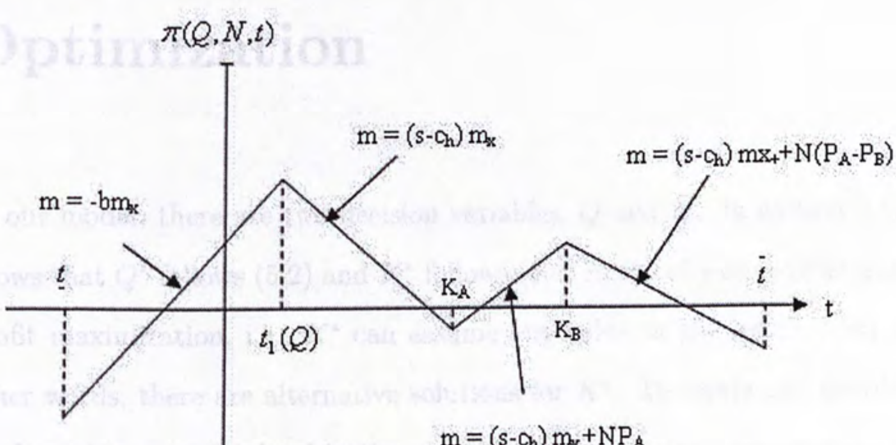

Figure 5.5: Total Newsvendor Profit, π against t with $K_A \geqslant t_1(Q^*)$.

Chapter 6

Objective1: Lexicographic Optimization

In our model, there are two decision variables, Q and N. In section 5.1, it shows that Q^* follows (5.2) and K^* follows (5.4) in the objective of expected profit maximization, i.e., K^* can assume any value in the set $[0, +\infty)$. In other words, there are alternative solutions for K^*. To tackle this problem, we formulate a multiple objective decision model for the newsvendor. We assume that the newsvendor ranks her objectives lexicographically, i.e., she ranks them in terms of importance and optimizes the objectives sequentially. The objective with the higher priority is the maximization of expected profit and the objective with the lower priority is the minimization of profit variance. That is, maximizing $E_t[\pi]$ has preemptive priority over minimizing $V_t[\pi]$. We denote this lexicographically optimization problem by

$$\text{lexicographic } [\max E_t[\pi(Q, N, t)], \min V_t[\pi(Q, N, t)]].$$

Thus, the newsvendor first maximizes $E_t[\pi(Q,N,t)]$ over the feasible set $F = \{(Q,N)^T | (Q,N)^T \geqslant 0\}$ and determines the optimal solution. If there is a unique solution to the maximization problem, the process ends and the variance is determined using the optimal solution for the decision vector $(Q,N)^T$. However, if the maximization of $E_t[\pi(Q,N,t)]$ results in alternative solutions (as will be the case in our problem), then $V_t[\pi(Q,N,t)]$ is minimized over the new feasible set defined by the multiple optimal solutions. For the use of lexicographic method in other multiple objective problems; see, for example, Ignizio [18, p.380] and Yu [31].

When the call spread is priced under (4.3), the expected profit with the call spread does not depend on the number of shares of call spread. This result will be useful in solving the newsvendor's lexicographic optimization since it will show that there exist alternative solution that maximize $E_t[\pi]$.

Proposition 2 *If the call spread is priced under (4.3). Then the optimal order quantity that maximizes the newsvendor's expected total profit with the call spread $E_t[\pi(Q,N,t)]$ is the same as the optimal order quantity that maximizes the newsvendor's expected profit without the call spread $E_x[\pi_1(Q,x(t))]$. Moreover, the optimal order quantity gives rise to the same maximum expected profit with or without the call spread.*

Proof:

$$\frac{d}{dQ}|_{Q=Q^*} E_t[\pi(Q,N,t)] = \frac{d}{dQ}|_{Q=Q^*} E_x[\pi_1(Q,x(t))] = 0$$

$$E_t[\pi(Q^*, N, t)] = E_x[\pi_1(Q^*, x(t))] + E_t[\pi_2(N, t)] = E_x[\pi_1(Q^*, x(t))]$$

Q.E.D.

This result demonstrates that the call spread does not change the newsvendor's expected profit. However, the call spread changes the risk profile of the newsvendor.

Remark 2 *Since the objective functions are identical for both problems (with or without the call spread), the problem of maximizing $E_t[\pi(Q, N, t)]$ results in alternative solutions. More specifically, the optimal solution is $(Q^*, N^*)^T$ where the optimal order quantity Q^* follows (5.2) and the optimal number of shares of call spread can assume any value in the interval $[0, \infty)$.*

To summarize, the newsvendor's lexicographic optimization problem is solved in two stages: In stage 1, she maximizes $E_x[\pi_1(Q, x(t)]$ and determine the optimal order quantity Q^* following (5.2). In stage 2, she minimizes $V_t[\pi(Q^*, N, t)]$ over the feasible set $\{N | N \geqslant 0\}$.

This chapter is organized in the following way: In section 6.1, we discuss the equivalence between the newsvendor's lexicographic objectives and her utility function. In section 6.2, we discuss the convexity condition and the solution procedure for stage 2. In section 6.3, some numerical examples will be carried out.

6.1 Equivalence between Lexicographic Optimization and Expected Utility Maximization

Before we commence the analysis of the optimization problems, it would be worthwhile to comment on the equivalence between the newsvendor's lexicographic objectives and her utility function. This result is presented in the following theorem.

Theorem 3 *Assume that the newsvendor optimizes her objectives* $[E[\pi], V[\pi]]$ *lexicographically. This is equivalently to maximize the newsvendor's expected utility when the utility function is quadratic.*

Proof: Recall from Proposition 2 and from Remark 2 that maximizing $E_t[\pi(Q, N, t)]$ gives optimal $(Q^*, N^*)^T$ where Q^* follows (5.2) and the N^* can assume any value in the interval $[0, \infty)$. Thus, the maximum expected profit with or without the portfolio has the same value, i.e., $E_t[\pi(Q, K, t)] = E_x[\pi_1(Q, x(t))]$.

Consider the case where the newsvendor is risk-averse and her utility function for profit π is quadratic given as $u(\pi) = a\pi - b\pi^2, a, b > 0$. Taking expectations, we find the expected utility can be expressed as

$$E_t[u(\pi)] = u(E_t[\pi]) - bV_t[\pi] \tag{6.1}$$

Therefore, maximizing the expected utility reduces to a tradeoff between the mean and the variance of profit. We will now show that $\pi(Q, N, t)$ stochastically dominates $\pi_1(Q, x(t))$, that is $E_t[u(\pi(Q, N, t))] > E_x[u(\pi_1(Q, x(t)))]$ iff $V_t[\pi(Q, N, t)] < V_x[\pi_1(Q, x(t))]$. First note that since $E_t[\pi(Q, N, t)] = E_x[\pi_1(Q, x(t))]$, neither $\pi(Q, N, t)$ nor $\pi_1(Q, x(t))$ dominates each other under FSD (first order stochastic dominance). Now, the difference $\Delta(\pi(Q, N, t), \pi_1(Q, x(t)))$ in the newsvendor's expected utility with vs. without the call spread is given by

$$\begin{aligned}
\Delta(\pi(Q, N, t), \pi_1(Q, x(t))) &\equiv E_t[\pi(Q, N, t)] - E_x[\pi_1(Q, x(t))] \\
&= u(E_t[\pi(Q, N, t)]) - u(E_x[\pi_1(Q, x(t))]) \\
&\quad -b[V_t[\pi(Q, N, t)] - V_x[\pi_1(Q, x(t))]] \\
&= -b[V_t[\pi(Q, N, t)] - V_x[\pi_1(Q, x(t))]].
\end{aligned}$$

Thus, when $V_t[\pi(Q, N, t)] - V_x[\pi_1(Q, x(t))]$, the random profit $\pi(Q, N, t)$ with the call spread stochastically dominates the random profit without the call spread $\pi_1(Q, x(t))$, i.e., we have $E_t[u(\pi(Q, N, t))] > E_x[u(\pi_1(Q, x(t)))]$. This means that lexicographic optimization is equivalent to maximization of the newsvendor's expected utility when shes uses the call spread. Q.E.D.

6.2 Minimizing the Conditional Profit Variance given Q^*

Before we begin to minimize the conditional profit variance, the convexity condition must be imposed in order to ensure the existence of optimal N such that the conditional variance given Q^* exists a global minimum. To prove the convexity of conditional profit variance, the following lemma is used.

Lemma 2 *A variance function $V_y[f(\hat{x},\tilde{y})]$ is convex in $\hat{x}$ if the linearity of f w.r.t. $\hat{x}$ given a fixed random variable $\tilde{y}$ holds.*

Before the proof of lemma 2. The definition of a linear function have to be explained.

Definition 1 *A function f is linear if and only if*

$$f(\sum_{i=1}^{n} a_i x_i) = \sum_{i=1}^{n} a_i f(x_i), \text{ for all } a_i \in \mathbb{R}$$

Proof of Lemma 2:

$$\forall x_1, x_2 \geqslant 0, \quad \lambda \in [0,1]$$

$$\begin{aligned} V_y[f(\lambda x_1 + (1-\lambda)x_2, y)] &= V_y[\lambda f(x_1,y) + (1-\lambda)f(x_2,y)] \text{ (linearity property of f)} \\ &= \lambda^2 V_y[f(x_1,y)] + 2\lambda(1-\lambda)cov[f(x_1,y), f(x_2,y)] \\ &\quad +(1-\lambda)^2 V_y[f(x_2,y)] \end{aligned}$$

Since

$$
\begin{aligned}
V_y[f(x_1,y)-f(x_2,y)] &\geqslant 0 \\
V_y[f(x_1,y)]+V_t[f(x_2,y)]-2cov[f(x_1,y),f(x_2,y)] &\geqslant 0 \\
\lambda(1-\lambda)V_y[f(x_1,y)]+\lambda(1-\lambda)V_y[f(x_2,y)] & \\
-2\lambda(1-\lambda)cov[f(x_1,y),f(x_2,y)] &\geqslant 0 \\
(\lambda^2-\lambda)V_y[f(x_1,y)]+[(1-\lambda)^2-(1-\lambda)]V_y[f(x_2,y)] & \\
+2\lambda(1-\lambda)cov[f(x_1,y),f(x_2,y)] &\leqslant 0 \\
\lambda^2V_y[f(x_1,y)]+2\lambda(1-\lambda)cov[f(x_1,y),f(x_2,y)] & \\
+(1-\lambda)^2V_y[f(x_2,y)] &\leqslant \lambda V_y[f(x_1,y)]+(1-\lambda)V_y[f(x_2,y)] \\
V_y[f(\lambda x_1+(1-\lambda)x_2,y)] &\leqslant \lambda V_y[f(x_1,y)]+(1-\lambda)V_y[f(x_2,y)]
\end{aligned}
$$

Q.E.D.

This lemma can be interpreted in this way. Given a random function $f(x,y)$ with a decision variable x and a random variable y. If $f(x,y)$ is linear in x for each realization of y, then the variance of $f(x,y)$ w.r.t. y will be convex in x. This lemma is applicable to our conditional profit function $V_t[\pi(Q^*,N,t)]$.

Since the decision variable Q^* is already solved in stage 1 and it becomes a constant. There is only one unsolved decision variable N^* remained in stage 2. Therefore, the conditional profit function $\pi(Q^*,N,t)$ is equivalent to $f(x,y)$ with $x=N$ and $y=t$.

Since $\pi(Q^*, N, t)$ is linear in N for each realization of t (refer to (5.9) and (5.13)). By lemma 2, $V_t[\pi(Q^*, N, t)]$ is convex in N. There exists a global minimum of conditional profit variance, denoted as V^*

To find the explicit solution of N^*, a direct way is to equate the first derivative of $V_t[\pi(Q^*, N, t)]$ w.r.t. N to zero and then solve the equation. However, it is analytically difficult to solve the equation. To find out the solution, an implicit way is to use exhaustive search method implemented in some computing softwares such as matlab.

6.3 Numerical Examples

6.3.1 Convexity of conditional profit variance

Suppose r = 0.5, T = 2, c_x = 250, m_x = -250, s = 25, c = 19, c_h = 15, b = 50, C_A = 6, C_B = 5.75, K_A = -0.2, K_B = 1.3, P_A = 3, P_B = 1.5, and t follows normal distribution with μ_t = -0.1. By varying the value of σ_t, it can be observed that the conditional profit variance $V_t[\pi(Q^*, N)]$ given the optimal order quantity Q^* is convex in the number of shares of call spread N. On the other hand, there exists a global minimum value of the conditional profit variance. The relationship between $V_t[\pi(Q^*, N)]$ and N are depicted in Figure 6.1, Figure 6.2, Figure 6.3 and Figure 6.4 with different values of σ_t.

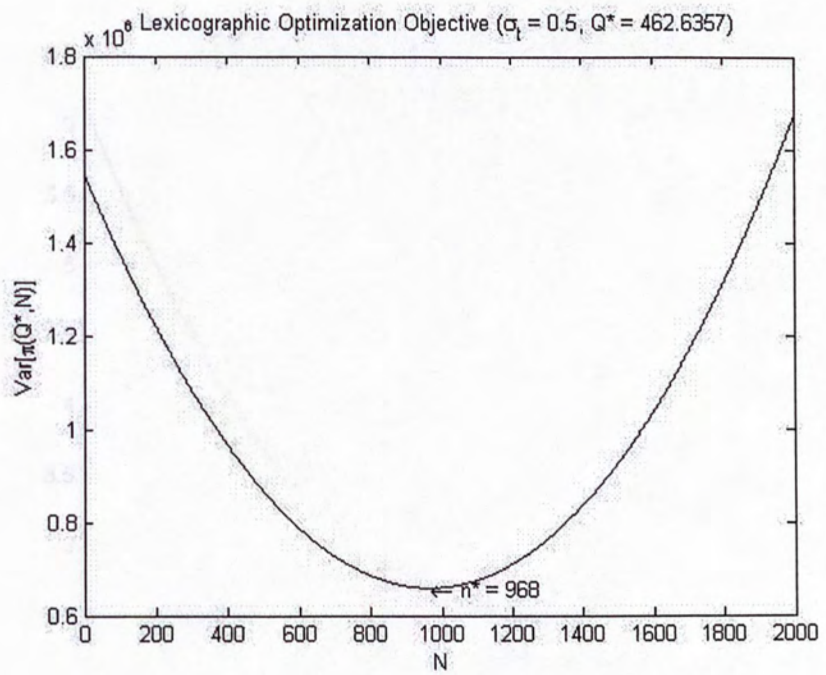

Figure 6.1: Conditional Profit Variance, $\sigma_t = 0.5$.

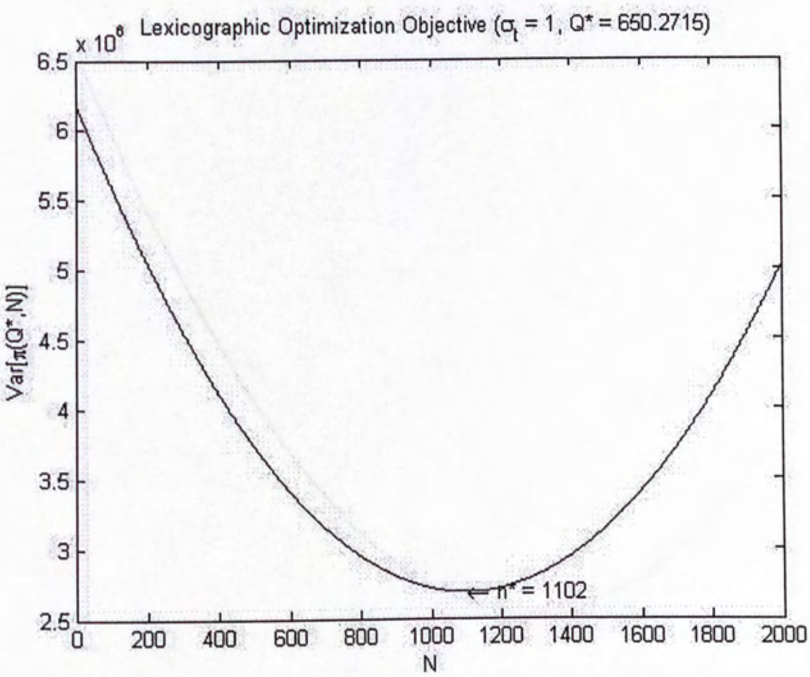

Figure 6.2: Conditional Profit Variance, $\sigma_t = 1$.

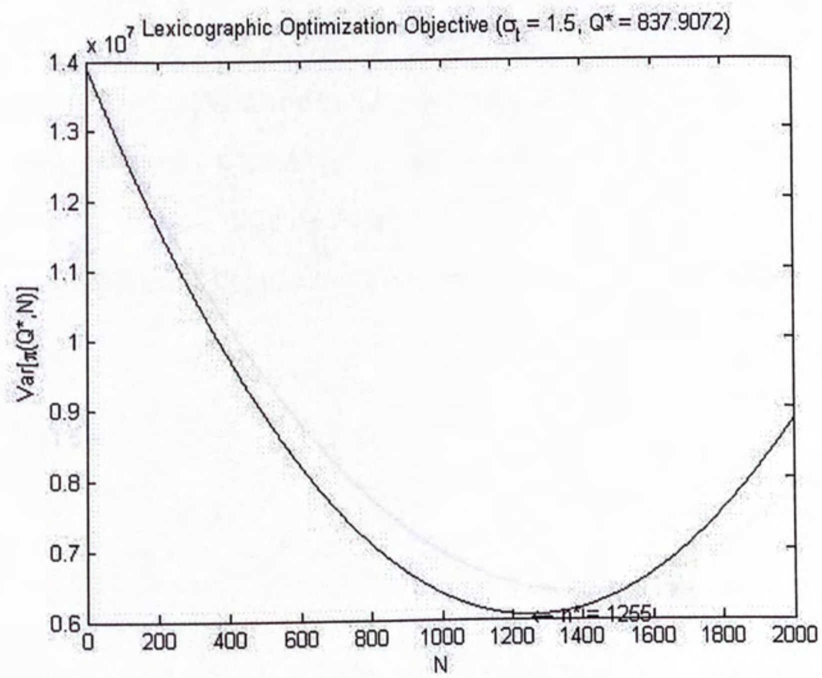

Figure 6.3: Conditional Profit Variance, $\sigma_t = 1.5$.

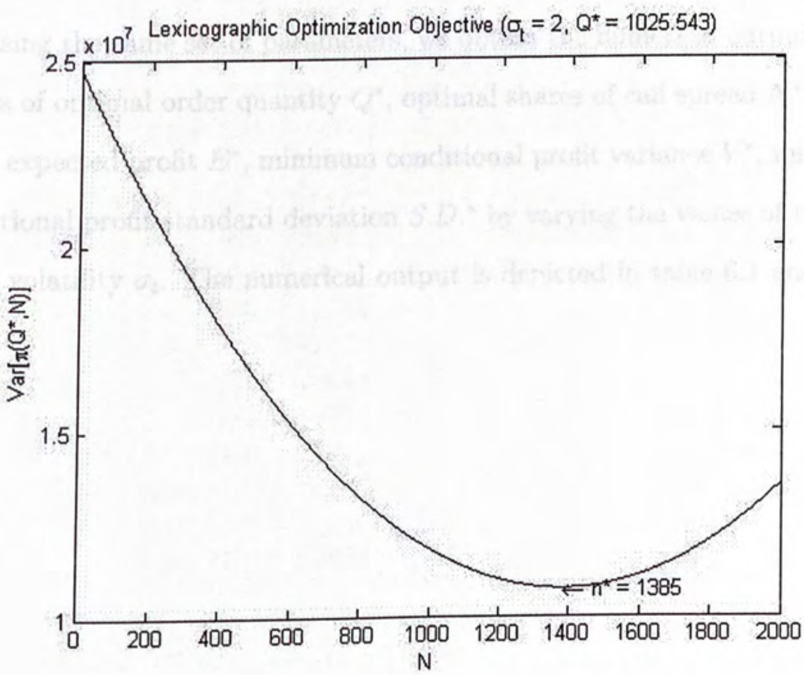

Figure 6.4: Conditional Profit Variance, $\sigma_t = 2$.

6.3.2 Correlation between Q^* & N^*

From the graphs above, besides the convexity, it can be observed that the optimal order quantity, Q^* and optimal shares of call spread, N^* increase concurrently with temperature index volatility, σ_t. It can be shown that Q^* and N^* are positively correlated in lexicographic optimization objective.

Using the same set of parameters, we obtain the numerical output of the values of optimal order quantity Q^*, optimal shares of call spread N^*, maximum expected profit E^*, minimum conditional profit variance V^*, minimum conditional profit standard deviation $S.D.^*$ by varying the values of temperature volatility σ_t. The numerical output is depicted in table 6.1 and table 6.2.

σ_t	Q^*	N^*	E^*	V^*	$S.D.^*$
0.1	313	723	1456	26005	161
0.2	350	857	1262	100750	317
0.3	388	914	1068	231060	481
0.4	425	945	874	417150	646
0.5	463	967	680	658850	812
0.6	500	988	486	955630	978
0.7	538	1013	292	1307000	1143
0.8	575	1040	98	1712900	1309
0.9	613	1070	-96	2173000	1474
1	650	1101	-290	2687100	1639
1.1	688	1132	-484	3255000	1804
1.2	725	1164	-678	3876100	1969
1.3	763	1195	-871	4550400	2133
1.4	800	1225	-1065	5277400	2297
1.5	838	1254	-1259	6057000	2461
1.6	875	1282	-1453	6889000	2625
1.7	913	1309	-1647	7773300	2788
1.8	950	1335	-1841	8709800	2951
1.9	988	1360	-2035	9698600	3114
2	1026	1384	-2229	10740000	3277
2.1	1063	1407	-2423	11833000	3440
2.2	1101	1428	-2617	12979000	3603
2.3	1138	1449	-2811	14177000	3765
2.4	1176	1469	-3005	15427000	3928
2.5	1213	1487	-3199	16730000	4090

Table 6.1: Optimal Decisions with different temperature volatility under lexicographic optimization ($\sigma_t = 0.1$ to 2.5)

σ_t	Q^*	N^*	E^*	V^*	$S.D.^*$
2.6	1251	1505	-3393	18086000	4253
2.7	1288	1522	-3587	19495000	4415
2.8	1326	1539	-3781	20957000	4578
2.9	1363	1554	-3975	22472000	4740
3	1401	1569	-4169	24040000	4903
3.1	1438	1583	-4363	25661000	5066
3.2	1476	1597	-4557	27336000	5228
3.3	1513	1610	-4751	29065000	5391
3.4	1551	1622	-4945	30846000	5554
3.5	1589	1634	-5139	32682000	5717
3.6	1626	1646	-5333	34571000	5880
3.7	1664	1657	-5527	36515000	6043
3.8	1701	1667	-5721	38512000	6206
3.9	1739	1677	-5914	40563000	6369
4	1776	1687	-6108	42669000	6532
4.1	1814	1696	-6302	44829000	6695
4.2	1851	1705	-6496	47043000	6859
4.3	1889	1714	-6690	49311000	7022
4.4	1926	1722	-6884	51634000	7186
4.5	1964	1730	-7078	54011000	7349
4.6	2001	1738	-7272	56443000	7513
4.7	2039	1745	-7466	58929000	7677
4.8	2076	1752	-7660	61470000	7840
4.9	2114	1759	-7854	64066000	8004
5	2151	1766	-8048	66716000	8168

Table 6.2: Optimal Decisions with different temperature volatility under lexicographic optimization ($\sigma_t = 2.6$ to 5.0)

The relationships between Q^* & N^* are illustrated by Figure 6.5 and Figure 6.6.

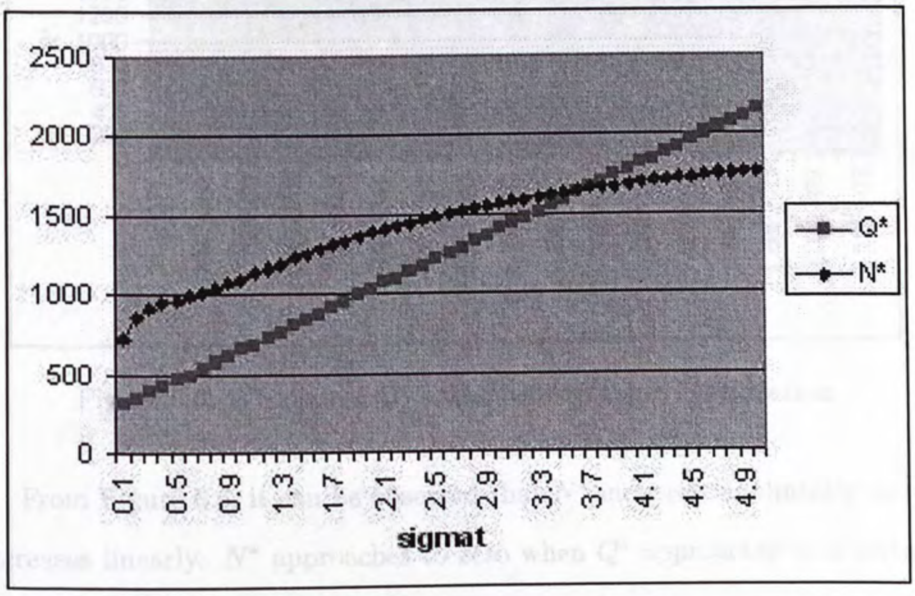

Figure 6.5: (Q^*, N^*) against σ_t under lexicographic optimization.

From Figure 6.5, it can be observed that Q^* increases linearly as σ_t increases. N^* increases nonlinearly as σ_t increases.

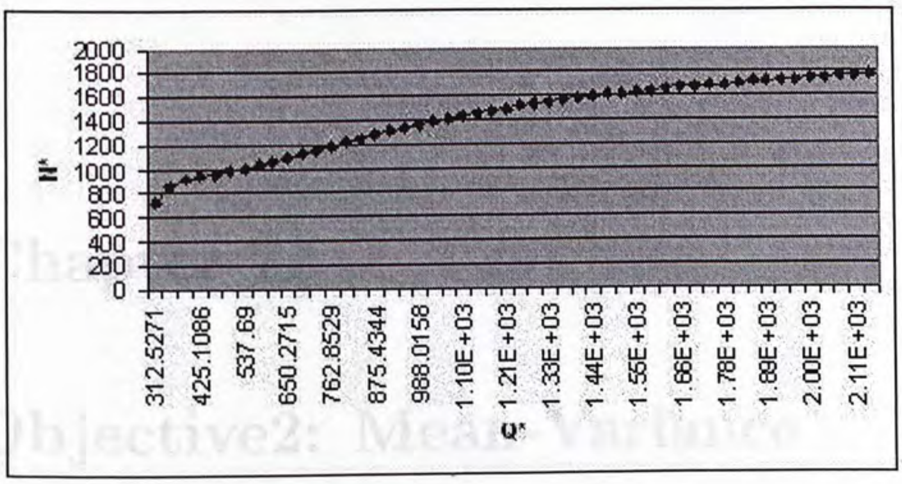

Figure 6.6: N^* against Q^* under lexicographic optimization.

From Figure 6.6, it can be observed that N^* increases nonlinearly as Q^* increases linearly. N^* approaches to zero when Q^* approaches to a certain level, Q_H, where $Q_H = K_A m_x + c_x = (-0.2) \times (-250) + 250 = 300$ in this example. The observation is consistent to (5.12). On the other hand, N^* approaches to a finite value, $\hat{N}$ when Q^* approaches to infinity.

$$\lim_{Q^* \to Q_H^+} N^* = 0 \tag{6.2}$$

$$\lim_{Q^* \to +\infty} N^* = \hat{N} \tag{6.3}$$

The analytical justification of (6.2) and (6.3) will be regarded as the future work.

Chapter 7

Objective2: Mean-Variance Optimization

As mentioned, all standard treatments of inventory models confine themselves to the optimization of the expected value of a given cost or profit measure, without consideration of any risk measures. The risk measures include the variance of profit/cost as well as that of the customer waiting times. This chapter analyzes the model, with variance of profit as the risk measure, exhibiting how the resulting inventory strategies differ from those obtained in the standard analysis.

We proceed with an analysis of the mean-variance tradeoff in this model.
(P1)

$$\max_{Q,N\geqslant 0} \quad E_t[\pi(Q,N,t)] - \beta V_t[\pi(Q,N,t)]$$

By taking the negative sign of the objective function with normalization, (P1) is transformed into (P2).

(P2)

$$\min_{Q,N\geqslant 0} \quad V_t[\pi(Q,N,t)] - \beta' E_t[\pi(Q,N,t)]$$

Using Lagrangian Multiplier Theory, (P2) can be transformed into (P3)

(P3)

$$\begin{aligned} \min_{Q,N\geqslant 0} \quad & V_t[\pi(Q,N,t)] \\ s.t. \quad & E_t[\pi(Q,N,t)] = \pi_T \end{aligned}$$

Note that π_T is a computed value dependent on β'.

Recall from (5.3), $E_t[\pi(Q,N,t)] = E_x[\pi_1(Q,t)]$, (P3) can be transformed into (P4)

(P4)

$$\begin{aligned} \min_{Q,N\geqslant 0} \quad & V_t[\pi(Q,N,t)] \\ s.t. \quad & E_x[\pi_1(Q,t)] = \pi_T \end{aligned}$$

Note that the target expected profit level must be smaller than or equal to the maximum expected profit, i.e. $\pi_T \leqslant E_x[\pi_1(Q^*,t)]$, where Q^* is given by (5.2). If this condition does not hold, the mean-variance optimization problem becomes infeasible.

(P1) is an unconstrained optimization problem of general mean-variance function (apart from non-negativity constraint). It is difficult to be optimized analytically due to multi-objective function and multi-dimension decision space. With the transformation from (P1) to (P4), it becomes constrained optimization problem with single-objective function and finite feasible solution set (due to concavity of the constraint equation). The transformation greatly reduces the computational effort in solving the optimization problem.

Before we commence the algorithm for solving the newsvendor problem under the mean-variance framework, it would be worthwhile to analyze the concavity of the mean-variance objective function.

Proposition 4 *The mean-variance function with maximization objective as shown in (P1) is not jointly concave in Q and N for some sets of values for operation, option, random parameters and risk aversion parameter, β.*

Proof: We show this proposition by a counter example. Suppose r = 0.5, T = 2, c_x = 250, m_x = -250, s = 33, c = 31, c_h = 10, b = 8, K_A = -0.2, K_B = 1.3, P_A = 3, P_B = 1.5, C_A = 1.1568, C_B = 0.2482, β = 4 and t follows normal distribution with μ_t = -0.1 and σ_t = 2.5. The relationship between MV value, Q^* and N^* is depicted in Figure 7.1. The relationship between MV value and Q^* for fixed N is depicted in Figure 7.2.

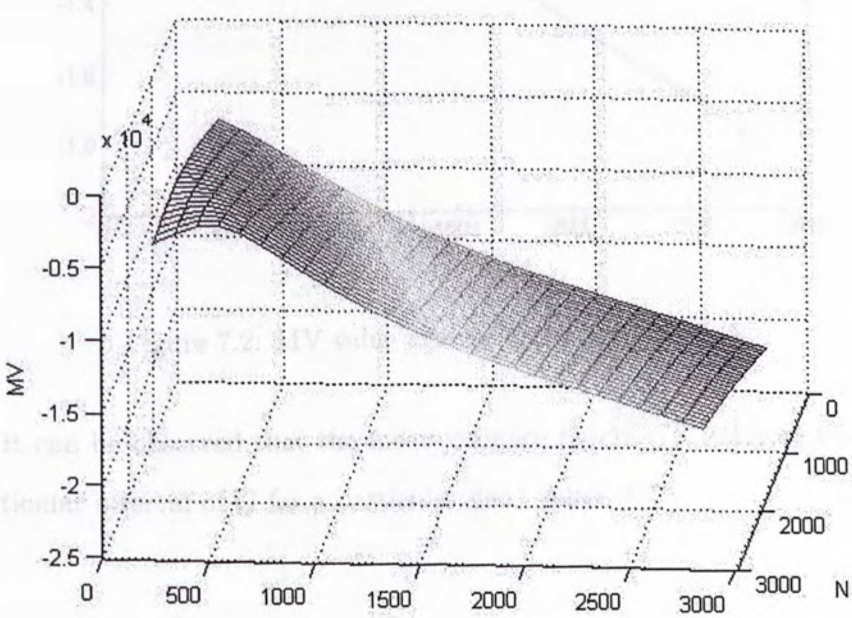

Figure 7.1: MV value against (Q, N).

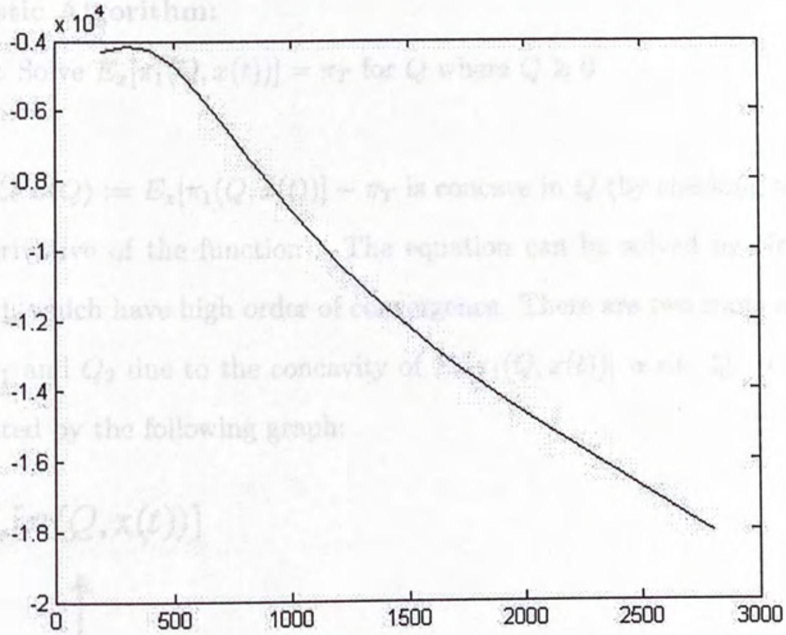

Figure 7.2: MV value against Q for some fixed N.

It can be observed that the mean-variance function is not concave for a particular interval of Q for a particular fixed value of N. Q.E.D.

Heuristic Algorithm:

Stage1: Solve $E_x[\pi_1(Q,x(t))] = \pi_T$ for Q where $Q \geqslant 0$

Since $a(Q) := E_x[\pi_1(Q,x(t))] - \pi_T$ is concave in Q (by checking the second derivative of the function). The equation can be solved by Newton's Method, which have high order of convergence. There are two roots at most $Q = Q_1$ and Q_2 due to the concavity of $E_x[\pi_1(Q,x(t))]$ w.r.t. Q. It can be illustrated by the following graph:

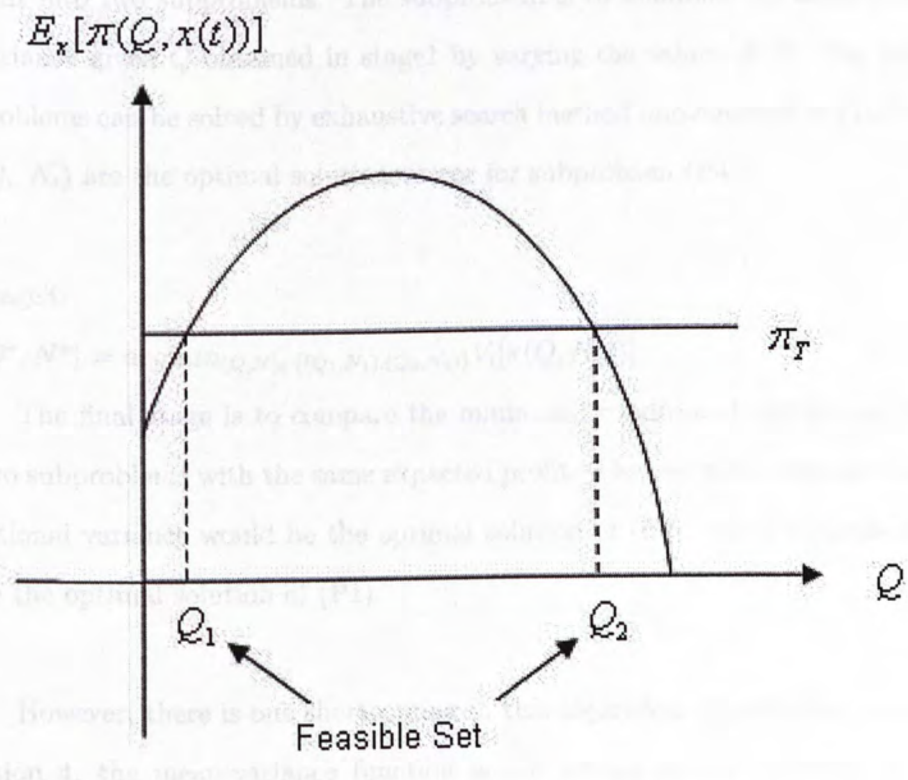

Figure 7.3: Feasible Q under MV framework.

Stage2: Two subproblems:

(P4.1):

$\min_{N \geqslant 0} \quad V_t[\pi(Q_1, N, t)]$

(Q_1, N_1) is optimal to the subproblem (P4.1)

(P4.2):

$\min_{N \geqslant 0} \quad V_t[\pi(Q_2, N, t)]$

(Q_2, N_2) is optimal to the subproblem (P4.2)

After retrieving the feasible solution set of Q in stage1. the problem is split into two subproblems. The subproblem is to minimize the conditional variance given Q obtained in stage1 by varying the values of N. The subproblems can be solved by exhaustive search method implemented in matlab. (Q_i, N_i) are the optimal solution vector for subproblem (P4.i).

Stage3:

$(Q^*, N^*) = argmin_{(Q,N) \in \{(Q_1,N_1),(Q_2,N_2)\}} V_t[\pi(Q, N, t)]$

The final stage is to compare the minimized conditional variance of the two subproblems with the same expected profit. The one with a smaller conditional variance would be the optimal solution of (P4), which equivalently be the optimal solution of (P1).

However, there is one shortcoming in this algorithm. Recall from proposition 4, the mean-variance function is not always jointly concave. It is uncertain that the algorithm can converge to the global optimum (Q^*, N^*) of the mean-variance function.

7.1 Numerical Examples

In this example, we let r = 0.5, T = 2, c_x = 250, m_x = -250, s = 25, c = 19, c_h = 15, b = 50, C_A = 6, C_B = 5.75, K_A = -0.2, K_B = 1.3, P_A = 1, P_B = 1, β = 0.5, and t follows normal distribution with μ_t = -0.1 and σ_t = 1 The following are the numerical output of the relationship between mean-variance function value and (Q, N) with call spread and without call spread respectively. Q starts from 100 to 1000, with an increment of 100. N starts from 0 to 9000, with an increment of 1000.

		N									
		0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Q	100	-13838	-13873	-13911	-13953	-13997	-14045	-14095	-14148	-14202	-1425[illegible]
	200	-9422	-9428	-9441	-9461	-9487	-9517	-9552	-9591	-9634	-9680
	300	-5737	-5694	-5667	-5652	-5649	-5656	-5671	-5694	-5723	-5757
	400	-2929	-2813	-2727	-2668	-2632	-2615	-2614	-2626	-2648	-2679
	500	-1264	-1033	-853	-727	-651	-615	-609	-625	-655	-697
	600	-824	-490	-200	12	125	157	142	102	48	-14
	700	-989	-630	-298	-25	145	205	198	158	103	39
	800	-1390	-1034	-708	-437	-262	-189	-186	-218	-269	-329
	900	-1790	-1434	-1108	-837	-662	-589	-586	-618	-669	-729
	1000	-2190	-1834	-1508	-1237	-1062	-989	-986	-1018	-1069	-1129

Table 7.1: MV value for different (Q, N) with call spread

(Q^*, N^*) = (700,5000), MV^* = 205.

							N				
		0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Q	100	-13838	-13838	-13838	-13838	-13838	-13838	-13838	-13838	-13838	-1383
	200	-9422	-9422	-9422	-9422	-9422	-9422	-9422	-9422	-9422	-9422
	300	-5737	-5737	-5737	-5737	-5737	-5737	-5737	-5737	-5737	-5737
	400	-2929	-2929	-2929	-2929	-2929	-2929	-2929	-2929	-2929	-2929
	500	-1264	-1264	-1264	-1264	-1264	-1264	-1264	-1264	-1264	-1264
	600	-824	-824	-824	-824	-824	-824	-824	-824	-824	-824
	700	-989	-989	-989	-989	-989	-989	-989	-989	-989	-989
	800	-1390	-1390	-1390	-1390	-1390	-1390	-1390	-1390	-1390	-1390
	900	-1790	-1790	-1790	-1790	-1790	-1790	-1790	-1790	-1790	-1790
	1000	-2190	-2190	-2190	-2190	-2190	-2190	-2190	-2190	-2190	-2190

Table 7.2: MV value for different (Q, N) without call spread

$Q^* = 600$, $MV^* = -824$,

From Table 7.1 and Table 7.2, it can be shown that the MV^* with call spread is larger than the one without call spread. On the other hand, the value of Q^* is larger in the case with call spread than without. This numerical result is significant because this implies a firm should order more products with a weather risk hedge contract than without in certain scenario.

Mathematically, the result is equivalent to

$$\begin{aligned} arg\max_{Q} f_\beta(Q,N) &> arg\max_{Q} f_\beta(Q,0) \\ \text{with domain set} &= \{(Q,N)^T \geqslant 0\} \end{aligned} \tag{7.1}$$

where $f_\beta(Q,N) := E_t[\pi(Q,N,t)] - \beta V_t[\pi(Q,N,t)]$

The analytical justification of this result is however left for the future studies.

Chapter 8

Conclusion and Future Work

In this thesis we attempt to design a hedging strategy to minimize the newsvendor non-catastrophic weather risk, the uncertainty in cash flow and earnings caused by weather volatility, or the financial exposure that a business may have to adverse weather events such as severe and continuous precipitation, rainstorms or typhoons. Through the employment of weather derivative, the newsvendor revenue stream can be stabilized.

We consider the models under lexicographic optimization and mean-variance framework. In lexicographic optimization, some promising results shows that the newsvendor risk can be partially hedged and the newsvendor expected utility is maximized with the employment of options. In mean-variance optimization, the mean-variance function is not always jointly concave in order quantity decision and hedging decision. Besides, a numerical result shows that the newsvendor mean-variance function value increases with the employment of options. On the other hand, a firm should order more products with a weather risk hedge contract than without.

The model can be extended to the optimization under VaR criterion. This is to assume that the risk-averse firm will choose to maximize the probability of exceeding a prespecified target profit level.

On the other hand, several modifications can be made on the original model.

1. General Demand Function:

   In the model, we assume that $x(t)$ and t are linearly correlated. General negatively-correlated demand function can be considered which reflect reality more.

2. Dependency of b, c_h on t:

   In the model, we assume that b and c_h are independent of t. b and c_h, in some sense, can be treated as related to t.

3. Multiple types of weather option contract:

   In the model, it is restricted that the number of shares of option A, N_A and option B, N_B longed and shorted respectively are the same. This constraint can be relaxed such that $N_A \neq N_B$ to diversify the types of weather hedging strategies.

After the model and analytic method are completely established, Other valuable extensions are worth to consider.

1. A Two Stage Problem:

   A compound option is an option to enter into an option, e.g., suppose that the initial premium of a compound option is \$0.2 million, which overs a period from Nov. 1 to March 31, with the first option expiration on Dec. 31. If the buyer decides to exercise that option, he pays a further \$0.8 million to enter into the option before Dec. 31. The buyer will be paid if the \$400K for each 0.1 degree, with a cap of \$4 million. A retailer may take the early sales information to update her demand forecast for the rest season and decides whether to complete the option buying, in conjunction with the stocking decision.

Interesting research issues include the value of the first option and how to integrate weather hedging and stocking decisions. This calls for an extension to the basic model.

2. Weather Risk Pooling:

   A US nationwide retailer of clothing and softgoods finds that sales fall off during adverse weather. Unexpected heat or cold at any time of the year lowers shoppers traffic in the short term and leads to unsold inventory over the course of a season. For example, a cool winter and early spring in Florida results in heavy mark-downs on bathing suits and resort wear, while a warm autumn in the Mid-West slows outerwear sales. Direct revenues fall while costs of inventory carry, promotion and sales rise. A financial company then offered the retail chain a contract based on a weather index across all the stores' regions (XL Weather & Energy (2004)).

   Thus, it is natural to extend our basic model to the multiple newsvendor setting with stock transhipment among vendors, That is, reallocation of initial ordered inventory during the season. This will be an interesting yet challenging problem.

Bibliography

[1] Baker, B. and N. Petrillo. 2004. Value of Weather Risk Management Contracts Reaches All-Time High. Available at http://www.wrma.org/Survey%20Results.html

[2] Boyabatl O. and L.B. Toktay. 2004. Operational hedging: A review with discussion. INSEAD, France. Working paper, e-mail: onur.boyabatliinsead.edu & beril.tokayinsead,edu

[3] Brody, D., Syroka J., Zervos M. 2002. Dynamic pricing of weather derivatives, Quantitative Finance. 2: 189-198

[4] Caldentey, R. and Haugh, M. 2003. Optimal control and hedging of operations in the presence of financial markets. Available at http://www.ssrn.com/abstract=447060

[5] Cao M.; Wei J. 2000. Pricing the weather. Risk Magazine. 67-70

[6] Chen, F. and A. Federgruen. 2000. Mean-variance analysis of basic inventory models. Technical report, Graduate School of Business, Columbia University , New York.

[7] Chen, Y. F. and M. Parlar. 2004. Value of a Put Option to the Newsvendor. Working paper, McMaster University.

[8] Cohen, M. A. and A. Huchzermeier. 1999. Global supply chain management: A survery of research and applications. In Tayur S., M. Magazine, R. Ganeshan (Eds.) Quantitative Models for Supply Chain Management. Kluwer Academic Publishers.

[9] Davis, Mark. 2001. Pricing weather derivatives by marginal value. Quantitative Finance. 1: 305-308

[10] Ding, Q. and P. Kouvelis. 2001. On the interaction of production and financial hedging decisions in global markets. Technical report, Washington University in St. Louis. (Second revision for Management Sci.)

[11] Dischel, B. 2001. Weather risk management at the Frozen Falls Fuel Company. Available at Chicago Mercantile Exchange's website: http://www.cme.com/products/index/weather.

[12] Dischel, B. and P. Barrieu. 2001. Weather hedging at the Hot Gas Company, a Case Study. Available at http://www.erivativesreview.com, a monthly magazine, July, v. 64.

[13] Figlewski, S; Levich, M. Richard. 2002. Risk management: the state of the art. Boston: Kluwer Academic.

[14] Gaur, V. and S. Seshadri. 2004. Hedging inventory risk through market instruments. Technical report, New York University, New York. (Revision of 2002 version.)

[15] German, H. 1999. Insurance and weather derivatives: from exotic options to exotic underlyings, Loudon : Risk

[16] Harrington, S. and G. Niehaus. 2003. United Grain Growers: Enterprise risk management and weather risk. Risk Manag. & Insurance Review. 6.2 (fall).

[17] Hull, J. 2002. Options, Futures, and Other Derivatives, 5th Ed. Prentice Hall, Signature.

[18] Ignizio, J. P. 1982. Linear Programming in Single and Multiple Objective Systems. Prentice-Hall, Englewood Cliffs, N.J.

[19] Jewson, S. and C. Rodrigo. 2002. The use of weather forecasts in the pricing of weather derivatives. Financial Economics Network, http://ssrn.com/abstract=405780

[20] Jewson, S. 2003. Closed-form expressions for the pricing of weather derivatives: Part 1 - The expected payoff. Financial Economics Network, http://ssrn.com/abstract=436262

[21] Li. Y. 2004. Weather Derivatives and their Applications in Hong Kong, Faculty of Business Adminstration, The Chinese University of Hong Kong

[22] Lustgarten, A. 2004. Banking on the weather. Fortune, May 31, 149 (11).

[23] Malinow, M. 2002. End-users (chapter 5). In Banks, E. (Ed.) Weather Risk Management. Palgrave, New York.

[24] Malinow, M. 2002. Case Study 1: Retailer. In Banks, E. (Ed.) Weather Risk Management. Palgrave, New York.

[25] Triantis, A. J. 2000. Real Options and corporate risk management. J. Applied Corporate Fin, 13(2).

[26] Van Mieghem, J. A. 2003. Capacity management, investment, and hedging: Review and recent developments. Manufacturing & Service Operations Management, 5 (4).

[27] WRMA, 2004. Weather Risk Management Association, http://www.wrma.org.

[28] XL Weather & Energy. 2004. Industries at Risk: Retail Available at http://www.xlwe.com/xlwe/xlwe_retail.htm.

[29] Ye. J. 2004. Hedging weather risks. The Sun (Newspaper, in Chinese), June 29.

[30] Yeh, J.H. and Y.F. Chow. 2004. Hedging Weather Risk: The Design, Pricing, and Applications to Hong Kong Disney. Inprint, Faculty of Business Administration, the Chinese University of Hong Kong.

[31] Yu, P. L. 1989. Multiple criteria decision making: Five basic concepts. In G. L. Nemhauser, A. H. G. Rinnooy Kan, and M. J. Todd, editors, Optimization, Handbooks in Operations Research and Management Science, pages 663-699. North-Holland, Amsterdam.

Appendix A

Weather Option Pricing

Assume that the temperature variable has zero systematic risk. (i.e. it has zero correlation with stock market returns.) From "Options, Futures, and Other Derivatives" by John C.Hull Section 28.3, it shows that the expected growth rate of a variable can reasonably be assumed to be the same in both the real world and the risk-neutral world if the variable has zero systematic risk. A actuarial approach can be used for valuing this weather derivative.

We could collect 50 years of data and estimate a probability distribution for the temperature. This in turn could be used to provide a probability distribution for the option payoff. Our estimate of the value of the option would be the mean of this distribution discounted at the risk free rate.

On the other hand, information updating can be performed. We might want to adjust the probability distribution for the temperature trends. For example, a linear regression might show that the cumulative February temperature is decreasing at a rate of 10 per year on average. The output from the regression can then be used to estimate a trend adjusted probability distribution for the temperature in same month next year.

Appendix B

Infeasibility of Perfect Hedge

According to Gaur and Seshadri (2001), if $D = c_x + m_x S$, then the newsvendor risk can be perfectly hedged.

In our model, if $x(t_T) = c_x + m_x t_T$, the realized profit at time T

$$
\begin{aligned}
\pi_T &= s\min\{x(t_T), Q\} + c_h \max\{Q - x(t_T), 0\} - b\max\{x(t_T) - Q, 0\} - cQ \\
&= -bm_x t_T + [(s+b-c)Q - bc_x] + [(s - c_h + b)m_x \max\{t_T - \frac{Q - c_x}{m_x}, 0\}]
\end{aligned}
$$

Provided that there exists a financial instrument A such that $s_t = h(t_t)$ for all t.

The hedging transactions at time 0 are:

1. Borrow and sell $b|m_x|$ units of the underlying asset A at the current price $h^{-1}(s_0)$.
2. Buy $(s - c_h + b)|m_x|$ temperature call options with exercise price $\frac{Q-c_x}{m_x}$ and

exercise date T

3. Borrow a sum of money equal to $[(s+b-c)-bc_x]e^{-rT}$ at the risk free rate to be repaid at time T.

The realized profit at time 0 is

$\pi_H = b|m_x|h^{-1}(s_0)+[(s+b-c)-bc_x]e^{-rT}-(s-c_h+b)|m_x|e^{-rT}E\max\{t_T-\frac{Q-c_x}{m_x},0\}$

where $E\max\{t_T-\frac{Q-c_x}{m_x},0\}$ is the fair price of the temperature call option.

There is no randomness in the above expression.

But note that there is no such an financial instrument exist in the derivative market (A is not a real option). The above transaction strategy does not work, that the newsvendor risk cannot be hedged totally.